Differentiation Techniques Calculus of a Single Variable X. Du

- Differentiation techniques can be useful if the standard rules are too complicated.
- **Implicit differentiation**:
  - Often used in situations when you have a curve $f(x, y) = k$ and cannot isolate *y*.
  - Use chain rule! - Remember to add $\frac{dy}{dx}$ when differentiating with respect to *y*.
  - Chain rule is the basis for implicit differentiation.
  - Solve for $\frac{dy}{dx}$ at the end of all computations.
  - Use with trigonometric identities to differentiate inverse trigonometric functions.
  - Use with the definition of *e* to differentiate logarithmic functions.
  - Use with inverse functions.
- **Logarithmic Differentiation**
  - Most handy when $f(x)$ contains many complicated factors and powers.
  - Take the natural log of both sides and simplify function by expanding.
- Divide out a function: If you have a function that is a quotient of polynomials, you can divide out the quotient to make it easier to evaluate instead of using quotient rule.
- Rewrite the function using logarithmic or exponential properties.
  - Logarithms: expand the logarithm and evaluate each term separately. Base change to *e* if necessary.
  - Exponents: Rewrite in terms of base *e*. Ex: $a^x = e^{x \ln a}$
- Rewrite the function using trigonometric identities and definitions.
  - This can be used to verify the derivatives of secondary trigonometric functions.
- Rewrite hyperbolic functions using exponents.
- Using permutations
  - This can be useful when performing a derivative on a polynomial repeatedly.
  - $\frac{d^n}{dx^n} x^m = \left({}^m P_n\right) x^{m-n}$
  - $\frac{d^n}{dx^n} x^n = n!$

Further notes:

- Logarithmic differentiation can be easily used to verify the product and quotient rules.
- The rules involving taking the n^{th} derivative on a polynomial involving permutations can be empirically observed and proven using mathematical induction.